УДК 62-97; 519.876.5
DOI https://doi.org/10.32838/2663-5941/2019.3-1/15

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THE RESEARCH OF THE MODEL DEFORMATION PROCESS OF ROTATING OBJECTS

The stress-strained state mathematical model of rotating objects with the complex geometric configuration is offered. The class of the model is chosen as an mathematical physics equations incorrect problem for the rectilinear, toroidal and conical areas. The model of the stressed state of objects is realized, the directions for further researches are revealed.

Key words: *rotating objects, incorrect problem, types of models, deformation, stress.*

Introduction. The rotating objects technical state with the complex geometric configuration is considered in this article, and the basic characteristics of these objects are evaluated. Let's define three basic approaches to its solution. For a simulated object, the initial characteristics are known, which include all design parameters and expected operating conditions. Determine the value of the parameters by solving a certain functional equation. For objects with known characteristics it is possible to determine the functions that determine the action of the most common force, physical, climatic and other factors – for example, operating pressure in the system, rotation mode, thermal characteristics, etc. Consider the most general and natural case that arises when solving control problems, or aspects of the technical state. At the initial moment of time (for example, at the beginning of operation), the initial characteristics of the object are known, and the results of a technical survey determine the characteristics of the object, which are actually changed during the operation of the initial characteristics. The problem of studying the technical state of rotating objects of complex geometric configuration with the corresponding initial conditions and determine the deformation process and the stressed state of the object is formulated.

The problem formulation. During the analysis of the technical state of rotating objects of complex geometric configuration, in addition to the experimental evaluation methods [1; 2] of the main characteristics, there are widely applied methods of mathematical modeling [3–5], the implementation of which is complicated by the nature of phenomena and processes, simulated for existing objects. According to the approach proposed in [6], any problem of managing the existing objects, or assessing its technical state, can have three main approaches to its solution.

1. Let the initial characteristics $\bar{x}$, which include all design parameters and expected operating conditions, of the simulated object be known. In this case, the values of the $\bar{y}$ parameters, in which $\bar{x}$ characteristics transmit during the operation, can be obtained by solving a certain functional equation:

$$\bar{y} = \mathrm{A}(V_1, V_2, \ldots V_n)\,\bar{x} \tag{1}$$

where $\mathrm{A}(V_1, V_2, \ldots V_n)$ is a certain operator that formalizes all influences $V_1 . V_2 . \ldots V_n$, acting on the object during the operation; its structure is known. In fact, (1) is an equation, or a system of equations of mathematical physics with correctly defined practical and initial conditions, for which the resolution (exact or approximate) exists and is uniquely determined, as evidenced by the corresponding mathematical calculations and results [7]. Problem (1) is usually solved at the design stage. In this case, the characteristics V_1, V_2, $\ldots V_n$ are clearly formalized. In terms of practical use, the problem (1) for assessing the technical condition or object management can only be used to obtain preliminary results on the project state of the object during operation for the most standard model values of parameters $V_1, V_2, \ldots V_n$, which are significantly different from the actual operating conditions $V_1(t), V_2(t)$, $\ldots V_n(t)$, which are functions of time. They often take into account the actual operating conditions, but in the vast majority of cases the structure and quantitative characteristics of these functions are unknown.

2. If $\bar{x}$ characteristics are known for the objects, as well as the functions $V_1(t), V_2(t), \ldots V_m(t)$ specify the action of the most common force, physical, climatic and other factors – for example, working pressure in the system, mode rotation, thermal characteristics, etc. The peculiarity of this approach is that $V_1(t), V_2(t), \ldots V_m(t)$, $m<n$ are known functions. In this case, the problem is written in the following form:

$$\bar{y} = A_t\left(V_1(t), \ldots V_m(t)\right)\bar{x} \tag{2}$$

where A_t differs from A, given in (1), by a simpler structure. In this case, the correspondences are also established for the system or equation (2), but the boundary and initial conditions are simpler than in (1). In some cases, the problem (2) takes into account changes in the spatial configuration of the object being studied.

The most common and natural case that arises when solving management problems or aspects of the technical state of the objects being studied is the following: $\bar{x}$ characteristics of the object are known at the initial moment of time (for example, at the beginning of operation), and $\bar{y}$ characteristics of the object, which are $\bar{x}$ characteristics, actually changed during the operation, are determined according to the results of technical examination. In real cases, $\bar{x}$ and $\bar{y}$ ε are not determined on the entire surface of the body being studied (it is often impossible), but only at some area (subset) of this object. In this case, the task of management or technical state of an object is formalized as follows: the following problem must be solved:

$$\begin{cases} \bar{y} = A \cdot \bar{x} \\ \bar{x}(\Omega) = \overline{x_B} \\ \bar{y}(\Omega) = \overline{y_B} \end{cases}, \tag{3}$$

where Ω is the part of the surface, in which $\overline{x_B}$ and $\overline{y_B}$ are given, A is an operator with an unknown structure, ($\overline{x_B}$ and $\overline{y_B}$ are parameters of the object state at the entire area that it occupies). Obviously, the problem (3) is incorrectly set; it is necessary to propose certain conditions of regularization for its solution [8]. In this case, the problem (3) is decomposed into the following tasks:

a) defining $\bar{x}$ *and* $\bar{y}$ on the entire area, occupied by the studied object by the known values $\overline{x_B}$ and $\overline{y_B}$;

b) defining the structure of the operator A to determine the nature of the forces and loads acting on the object being investigated and its quantitative characteristics.

Problem solving:

a) with the choice of interpolation or data approximation apparatus, which allows you to reproduce $\bar{x}$ and $\bar{y}$ ε for the known $\overline{x_B}$ and $\overline{y_B}$. In this case, the device of cubic spline interpolation or the same splines with smoothing as well as the approximation device depending on information on the measurement accuracy $\overline{x_B}$ and $\overline{y_B}$ or on the nature of the change in the value $\bar{x}$ and $\bar{y}$ during the operation (linearity of dependencies, allocation of the most characteristic forces and loads, temperature regimes, etc.).

In the stage b), the structure of the operator A is renewed based on the defined values of $\bar{x}$ and $\bar{y}$. For example, the model of the deformation process and the stressed state is chosen, then the problem of type (2) is solved, and based on the known $\bar{x}$ there are defined:

$$\overline{y_t} = A_t^*\left(V_1(t), \ldots V_m(t)\right)\bar{x}\ . \tag{4}$$

The choice of A_t operator is considered to be complete if $\overline{y_t}$ value differs little from $\bar{y}$. According to the calculated operator:

$$A_t = \lim_{T \to N} A_t^*\ ,$$

The result of choosing A_t^* during the N-fold solution of the problem (4) is determined not only by $\bar{y}$ value but also by the predicted values of $\overline{y_B}$, which make it possible to make predictions for estimating the residual resource of the object.

The proposed approaches can be illustrated in the following example. During the study of the geometrical configuration of rotary kilns in the enterprises manufacturing cement, drying equipment in the sugar industry, etc., a simplified scheme of the mentioned constructions can be presented as a combination of contiguous rectilinear conic and toroidal areas:

a) straight-conical conjugation;

b) rectilinear-toroidal conjugation.

Let the coordinates of some set of points be specified at the control and the initial moments of time on the surface of the objects under study. Using interpolation procedures and methods of differential geometry, one can get an idea for three types of elements of the studied bodies construction:

– at the initial moment of time the radius vector of any point for a straight line area is written as:

$$\bar{V}_o = \begin{cases} x = s \\ y = r\cos\varphi \\ z = r\sin\varphi \end{cases}. \tag{5}$$

S is a longitudinal coordinate, S∈ $[S_o, S_n]$, where S_o is the initial, and S_n is the end point of the rectilinear axis, φ∈[0;2≠] is the polar angle; V=[R_1, R_2], R_1 is the internal, and R_2 is the external radii for the control moment of time representation (5) is transformed into the following representation:

$$\bar{r}_k = \begin{cases} x = s + \left(\alpha_n(s)\cdot sin\varphi + \alpha_b(s)cos\varphi\right)\rho(s,r,\varphi) \\ y = y(s) + \left(\beta_n(s)sin\varphi + \beta_b(s)cos\varphi\right)\rho(s,r,\varphi), \\ z = z(s) + \left(r_n(s)sin\varphi + r_b(s)cos\varphi\right)\rho(s,r,\varphi) \end{cases} \quad (6)$$

where $\rho(s,r,\varphi)$ is a function that defines the law of changing the radius of the area, $\{s, y(s), z(s)\}$ are the coordinates of the deformed axis of the area; the functions y(s) and z(s) are the results of implementation of the interpolation procedures [6], $\{\{\alpha_n(S), \beta_n(s), r_n(s)\}$ and $\{\alpha_b(S), \beta_b(s), r_b(s)\}$ are the coordinates of the unit vectors of the normal and the binormal to the pipeline axis, which are determined according to known formulas of differential geometry [9], the representation for the radius vector at the initial moment of time for the conical area is written as follows:

$$\bar{V}_o = \begin{cases} x = s \\ y = \rho(r,s)cos\varphi, \\ z = \rho(r,s)sin\varphi \end{cases} \quad (7)$$

where $\rho(r,s) = \frac{R_0 - R_1}{S_0 - S_1}s + \frac{R_1S_0 - R_0S_1}{S_0 - S_1} + v$

R_0 and R_1, respectively, are the largest and smallest radii of the conical area;

S_0 and S_1 are the initial and final longitudinal coordinates of the conical area;

r ∈ [0;δ]; δ is the thickness of the wall. At the control moment of time, the dependence (7) has the following form:

$$\bar{V}_k = \begin{cases} x = s + \left(\alpha_n(s)\cdot sin\varphi + \alpha_b(s)cos\varphi\right)\rho(s,r) \\ y = y(s) + \left(\beta_n(s)sin\varphi + \beta_b(s)cos\varphi\right)\rho(s,r), \\ z = z(s) + \left(r_n(s)sin\varphi + r_b(s)cos\varphi\right)\rho(s,r) \end{cases} \quad (8)$$

where, $\{s; y(s); z(s)\}$; $\{\alpha_n(s); \beta_n(s); r_n(s)\}$; $\{\alpha_b(s); \beta_b(s); r_b(s)\}$ are the components of the axis points of the deformed conical area, the normal and the binormal to it, respectively, $\rho(s,r)$ is determined similarly to (7). If the radii R_0, R_1 and coordinates S_0, S_1 1 change during the deformation process, these values are substituted in expression (7) for $\rho(s,r)$. For determining the coordinates $\{s, y(s), z(s)\}$ it is expedient to use the technique of Hermite polynomials in representation (8) on the condition that:

$$H(s_0) = \varepsilon_0 \qquad H(s_1) = \varepsilon_2$$

$$H^{\cdot}(s_0) = \varepsilon_1 \qquad H^{\cdot}(s_1) = \varepsilon_3, \quad (9)$$

the experimentally determined values of ε ε_0, ε_1, $\varepsilon_2, \varepsilon_3$, as a rule, satisfy the condition:

$$|\varepsilon_i| \ll 1, i = 1,2,3,4. \quad (10)$$

In addition, ε_0 and ε_1 must meet the conditions for conjugation of the rectilinear and conical areas. Under the conditions (9), the polynomial takes the following form:

$$H(s) = \varepsilon_1\left[(s - s_1)^2\left(\frac{2S}{(S_0 - S_1)^3} + \frac{-S_1 + 3S_0}{(S_0 - S_1)^3}\right)\right] +$$

$$+\varepsilon_2\left[\frac{(S - S_0)(S - S_1)^2}{(S_0 - S_1)^2}\right] + \varepsilon_3\left[(s - s_0)^2\left(\frac{-2S}{(S_1 - S_0)^3} +\right.\right.$$

$$\left.\left.+\frac{3S_1 - S_0}{(S_1 - S_0)^3}\right)\right] + \varepsilon_4\left[\frac{(S - S_0)^2(S - S_1)}{(S_1 - S_0)^2}\right]; \quad (11)$$

– for a toroidal area at an initial moment of time:

$$\bar{V}_0 = \begin{cases} x = (R_0 + rcos\varphi)cos\theta \\ y = (R_0 + rcos\varphi)sin\theta \\ z = rsin\varphi, \ \varphi \in [0; 2\pi]; \end{cases} \quad (12)$$

r is $[R_1; R_2]$ and θ is $[\varphi_1; \varphi_2]$, where R_1 and R_2 respectively, are the internal and external radii of the toroidal area; R_0 is its radius of curvilinear; φ_1 and φ_2 are the angles of opening the toroidal area ($\varphi_1 = 0$; $\varphi_2 = \frac{\pi}{2}$). All of these parameters are known from the design documentation; the representation (12) has the following form at the control point of time:

$$\bar{V}_k = \begin{cases} x = (R_H + rcos\varphi)cos\theta \\ y = (R_H + rcos\varphi)sin\theta \\ z = rsin\varphi, \ \varphi \in [0; 2\pi], \end{cases} \quad (13)$$

where R_H is a new radius of curvilineum of a toroidal area. According to the coordinates of the points $A_1(x_1, y_1, z_1), A_2(x_2, y_2, z_2), A_3(x_3, y_3, z_3$ (Fig. 1), the locus of the points, equally distant to A_1, A_2 and A_3, from the following system [6].

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 \\ (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 \end{cases}. \quad (14)$$

System (14) is a parametric equation of a straight line, which can be written after transformations as follows:

$$\begin{cases} x = x_1 + \alpha t \\ y = y_1 + \beta t \\ z = z_1 + \gamma t \end{cases} \quad (15)$$

Where (15) is the direction vector of a straight line, which is determined from (14). The equation of a plane passing through the points (A_1, A_2, A_3) is written as follows:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (16)$$

Substituting the relation (15), taking into account (13) in equation (16), we find the coordinates (x^*, y^*, z^*) of the point of intersection of the plane (16) with the straight line (15), and then:

$$R_H^2 = (x^* - x_1)^2 + (y^* - y_1)^2 + (z^* - z_1)^2 \quad (17)$$

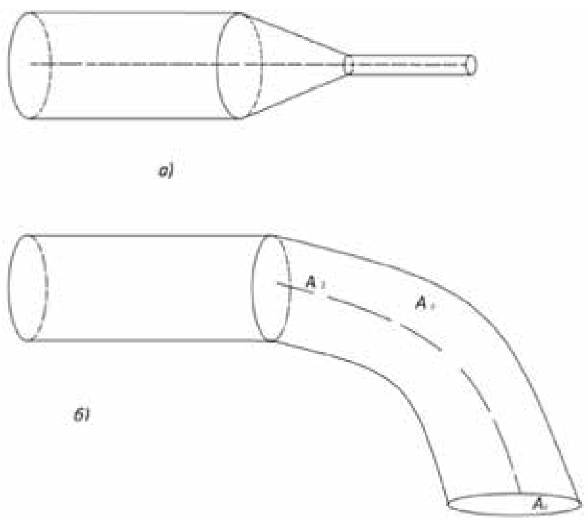

Fig. 1. Circuits of elements conjugation of rotating objects construction

The presentations (5–6), (7–8) and (12–13) fully describe the undeformed and deformed object of the study, and then one can assess the change in the stress-state state of the object within the framework of the theory of elasticity according to the known method [7]:

1. There are calculated components of the vectors of local bases for three types of areas according to formulas (5)–(6) – for rectilinear, (7)–(8) – for conic and (12)–(13) – for toroidal areas:

$$\bar{Э}_i^o = \frac{\partial \bar{r}_0}{\partial x_i}, x_1 = v; x_2 = \varphi; x_3 = s \qquad (18)$$

for rectilinear and conical areas:

$$\bar{Э}_i^o = \frac{\partial \bar{r}_0}{\partial x_i}, x_1 = \gamma; x_2 = \varphi; x_3 = \theta. \qquad (19)$$

at the initial moment and:

$$\bar{Э}_i^k = \frac{\partial \bar{r}}{\partial x_i} \qquad (20)$$

at a control moment of time.

2. The components of the matrix tensor are calculated as follows:

$$g_{ij}^0 = \bar{Э}_i^o \cdot \bar{Э}_i^o$$
$$g_{ij}^\kappa = \bar{Э}_i^\kappa \cdot \bar{Э}_J^o \qquad (21)$$

3. Then the components of the deformation tensor are calculated:

$$\varepsilon_{IJ} = \frac{1}{2}\left(g_{ij}^\kappa - g_{ij}^0\right) \qquad (22)$$

4. Then the components of the stress tensor are calculated in the framework of the linear theory of elasticity:

$$\sigma_{IJ} = \lambda \mathrm{I}_r(\varepsilon) g_{IJ} + 2\mu\varepsilon_{IJ} \qquad (23)$$

Where $\lambda \, and \, \mu$ are the parameters of the Lame material, $\mathrm{I}_r(\varepsilon)$ is the first invariant of the deformation tensor:

$$\mathrm{I}_r(\varepsilon) = \sum_{i=1}^{3}\sum_{j=1}^{3}\varepsilon_{ij} g^{ij} \qquad (24)$$

where g^{ij} are the components of the matrix, inverse to $\{g_{IJ}\}$, calculated according to (21) for the initial moment of time. Thus, the initial model of the process of deformation and the tense state of objects of complex geometric shape, which operate in conditions of rotation around one of the axes (usually-longitudinal), is fully developed.

Conclusions. We have developed an initial model for the process of deformation and stressed objects of complex geometric form, which operate in rotational conditions around one of the axes (generally, longitudinal):

– investigated the process of exploitation of objects and determined that the possible change in the process of operation depends on the initial conditions and the geometric shape of the object;

– the obtained results of the complex geometric configuration rotating objects work modeling were obtained by the method of data configuration..

References:

1. Неразрушающий контроль и диагностика : справочное пособие. / Под редакцией В.В. Клюева. Москва : Машиностроение. 2003. 654 с.
2. Вологжанинов Ю.И. Приближенные методы разделения напряженной и экспериментальной механике. Киев : Наукова думка. 1993. 157 с.
3. Алфутов Н.А. Основы расчета на устойчивость упругих систем. Москва : Машиностроение. 1991. 336 с.
4. Федоренко Д.П. Приближенные решения задач оптимального управления. Москва : Наука 1978. 478 с
5. Писаренко Г.С. Справочник по сопротивлению материалов. / Г.С. Писаренко, А.Л. Яковлев, В.В. Матвеев. Киев : Наукова думка. 1975. 704 с.
6. Олійник А.П. Математичний апарат для контролю НДС трубопроводів при зміні їх просторового положення. /А.П. Олійник, Л.М. Заміховський. Івано-Франківськ, ІФНТУНГ, 2008. 306 с.
7. Седов Л.И. Механика сплошных сред. Москва : Наука 1984. 572 с.
8. Тихонов А.Н. Методы решения некорректных задач. /А.Н. Тихонов, В.Я. Аренив. Москва : Наука 1979. 285 с.
9. Победря Б.Е. Лекции по тензорному анализу. Издательство МГУ. 1986. 264 с.

ДОСЛІДЖЕННЯ ПРОЦЕСУ ДЕФОРМУВАННЯ МОДЕЛІ ОБЕРТОВИХ ОБ'ЄКТІВ

Пропонується математична модель процесу деформування та напруженого стану протяглих обертових об'єктів складної геометричної конфігурації. Обрано клас моделі як некоректної задачі рівнянь математичної фізики. Запропоновано модель процесу деформування прямолінійної тороподібної та конічної ділянок. Запропоновано модель напруженого стану об'єктів, виявлено напрями подальших досліджень.

Ключові слова: *обертові об'єкти, некоректна задача, типи моделей, деформація, напруження.*

ИССЛЕДОВАНИЕ ПРОЦЕССА ДЕФОРМИРОВАНИЯ МОДЕЛИ ВРАЩАЮЩИХСЯ ОБЪЕКТОВ

Предлагается математическая модель процесса деформирования и напряженного состояния протянутых вращающихся объектов сложной геометрической конфигурации. Избран класс модели как некорректной задачи уравнений математической физики. Предложена модель процесса деформирования прямолинейных торообразных и конических участков. Предложена модель напряженного состояния объектов, выявлены направления дальнейших исследований.

Ключевые слова: *вращающиеся объекты, некорректная задача, типы моделей, деформация, напряжение.*